Published by Institute of Physics Publishing for SISSA

Received: *August 25, 2007*
Accepted: *September 28, 2007*
Published: *October 9, 2007*

Electrified BPS giants: BPS configurations on giant gravitons with static electric field

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Abstract: We consider D3-brane action in the maximally supersymmetric type IIB plane-wave background. Upon fixing the light-cone gauge, we obtain the light-cone Hamiltonian which is manifestly supersymmetric. The 1/2 BPS solutions of this theory (solutions which preserve 16 supercharges) are either of the form of spherical three branes, the giant gravitons, or zero size point like branes. We then construct specific classes of 1/4 BPS solutions of this theory in which static electric field on the brane is turned on. These solutions are deformations about either of the two 1/2 BPS solutions. In particular, we study in some detail 1/4 BPS configurations with electric dipole on the three sphere giant, i.e. BIons on the giant gravitons, which we hence call BIGGons. We also study BPS configurations corresponding to turning on a background uniform constant electric field. As a result of this background electric field the three sphere giant is deformed to squashed sphere, while the zero size point like branes turn into circular or straight fundamental strings in the plane-wave background, with their tension equal to the background electric field.

Keywords: D-branes, Penrose limit and pp-wave background, AdS-CFT Correspondence.

Contents

1. Introduction

The idea that a non-perturbative formulation of string theory dynamics can be obtained from (some particular limits of) D-brane dynamics has proved very fruitful; the BFSS matrix model [1] and the AdS/CFT duality [2] are indeed outcomes of this viewpoint. To exploit this idea further one needs to have a much better grasp of the brane dynamics than what we have now.

JHEP10(2007)043

One of the first steps in this direction was taken in [3] trying to reproduce the Polchinski's picture for D-branes [4], namely open strings with Dirichlet boundary conditions ending on the brane, from the D-brane theory. The low energy effective field theory of a single Dp-brane is a $p+1$ supersymmetric U(1) gauge theory with 16 supercharges which is described by the Dirac-Born-Infeld action plus the Chern-Simons terms through which brane couples to background RR form fields. At first order in α', i.e. at the quadratic action level, the theory reduces to supersymmetric $p+1$ dimensional Maxwell theory. In the supersymmetric Maxwell limit, it was shown that a BPS configuration corresponding to an electric charge on the p dimensional brane, which was called a "spike", correctly reproduces the behavior one expects from the open strings ending on branes [3]. This analysis was also extended to the full Born-Infeld action and the "spikes" were hence called BIons [3, 5].

The above mentioned analysis was mainly performed for D-branes residing in a background flat space. The $AdS_5 \times S^5$ and the ten dimensional plane-wave background, which can be obtained from the former by taking the Penrose limit [6], are of special interest both because they are the only maximally supersymmetric type IIB backgrounds and more importantly because of the AdS/CFT duality. The $AdS_5 \times S^5$ background allows 1/2 BPS spherical D3-brane configurations, the giant gravitons [7]. It was also shown that the plane-wave background also admits similar 1/2 BPS spherical 3-brane configurations e.g. see [8].

Analyzing BPS states, due to protection by supersymmetry, has been one of the most instructive ways in understanding and checking the AdS/CFT or plane-wave/CFT duality. Among these BPS states many of them could be related to deformations of a 1/2 BPS giant graviton state. These deformations can come in two classes, one in which the spherical shape of the giant has been deformed (e.g. see [9]) or those which besides the shape we have also turned on an electromagnetic gauge field on the brane (e.g. see [10 − 12]). In this paper we focus on the analysis of the second class and in particular the cases involving the static electric fields on the giant and classify all 1/4 BPS configurations for a given static electric field.

This paper is organized as follows. In section 2, we start with the D3-brane action on the plane-wave background. Fixing the light-cone gauge and the κ-symmetry we obtain the full supersymmetric light-cone Hamiltonian of the D3-brane in the plane-wave background. We also present the complete superalgebra of the Hamiltonian. In section 3, we study several 1/4 BPS configurations involving various static electric fields. In the absence of background electromagnetic fields, we have 1/2 BPS configuration which are either of the form of finite size spherical 3-brane, the giant gravitons, or a point like 3-brane, a spherical brane of zero size. In particular we study the case where the electric field is sourced by an electric dipole on the three sphere giant, i.e. the counterpart of BIons on the Giant Gravitons, which will hence be called BIGGons. This generalizes the Giant Hedgehogs of [11] to the full Born-Infeld theory. We show that the BIGGons, unlike their flat brane counterparts, have a finite extent and are not stretched to infinity. We also study configurations with constant electric field, showing that the electric field, similarly to the magnetic field [12], deforms (squashes) the three sphere giant. As a result of background electric field on the point like brane configurations, the brane behaves as a fundamental string on the plane-wave background with tension equal to the electric field. Section 4 contains our concluding remarks and outlook. Four appendices are added to fix our fermionic notation, introduce the "Polyakov form" of the DBI action, present some details of light-cone gauge fixing and analysis of BPS equation.

2. D3-brane light-cone Hamiltonian in the plane-wave background and its supersymmetry algebra

In this section we work out explicit form of the D3-brane light-cone Hamiltonian on the maximally supersymmetric type IIB plane-wave background. The plane-wave geometry is given by

$$\begin{aligned} ds^2 &= -2dX^+dX^- - \mu^2(X^iX^i + X^aX^a)(dX^+)^2 + dX^idX^i + dX^adX^a \\ C_{+ijk} &= -\frac{\mu}{g_s}\epsilon_{ijkl}X^l \quad , \quad C_{+abc} = -\frac{\mu}{g_s}\epsilon_{abcd}X^d \\ e^{\phi} &= g_s = constant \end{aligned} \tag{2.1}$$

where $i, a = 1, 2, 3, 4$ and C is the four-form potential of the self-dual five-form of type IIB. We have chosen our coordinates to make manifest the $\mathrm{SO}(4)\times\mathrm{SO}(4)$ symmetry of the transverse directions labeling vectors of the two SO(4)'s with i, a, as well as the translation symmetry in X^+ and X^- directions. In the above metric $\frac{\partial}{\partial X^-}$ is a (globally defined) null Killing direction and $\frac{\partial}{\partial X^+}$ is time like. For a more detailed discussion on the isometries of the background we refer the reader to [13].

2.1 Supersymmetric D3-brane action in the plane-wave background

The low energy supersymmetric effective action for a D3-brane in the plane-wave background is [14]

$$S = -\frac{1}{g_s}\int d^4\sigma\sqrt{-det\big(g_{\hat\mu\hat\nu} + F_{\hat\mu\hat\nu} + M_{\hat\mu\hat\nu}\big)} + \int \mathcal{L}_{WZ} \tag{2.2}$$

where

$$\begin{aligned} g_{\hat\mu\hat\nu} &= -2\partial_{\hat\mu}X^+\partial_{\hat\nu}X^- - \mu^2(X^I)^2\partial_{\hat\mu}X^+\partial_{\hat\nu}X^+ + \partial_{\hat\mu}X^I\partial_{\hat\nu}X^I \\ M_{\hat\mu\hat\nu} &= 2i\partial_{(\hat\mu}X^+(\bar\psi\bar\gamma^-\partial_{\hat\nu)}\psi + \psi\bar\gamma^-\partial_{\hat\nu)}\bar\psi) - 4\mu\bar\psi\bar\gamma^-\Pi\psi\partial_{\hat\mu}X^+\partial_{\hat\nu}X^+ \\ F_{\hat\mu\hat\nu} &= \partial_{\hat\mu}A_{\hat\nu} - \partial_{\hat\nu}A_{\hat\mu} - 2i\partial_{[\hat\mu}X^+(\psi\bar\gamma^-\partial_{\hat\nu]}\psi + \bar\psi\bar\gamma^-\partial_{\hat\nu]}\bar\psi) \end{aligned} \tag{2.3}$$

The hatted Greek indices are used for the worldvolume coordinate ranging over $0, 1, 2, 3$. The capital Latin indices, $I, J, \cdots$ are used to denote the eight transverse directions, that is $I = (i, a)$, in particular, X^I denote the eight transverse embedding coordinates of the brane. The $A_{\hat\mu}$ is the $\mathrm{U}(1)$ gauge field on the brane. Here we have set $2\pi\alpha' = 1$ and powers of α' can be recovered once needed, through dimensional analysis. The parenthesis in the expression for $M_{\hat\mu\hat\nu}$ means symmetrization on indices and hence $M_{\hat\mu\hat\nu}$ is symmetric, $M_{\hat\mu\hat\nu} = M_{\hat\nu\hat\mu}$. ψ's are the sixteen component complex but chiral fermions of type IIB. Note that the expressions in (2.3) have been written after fixing the κ-symmetry as [14]

$$\bar\gamma^+\psi = \bar\gamma^+\bar\psi = 0. \tag{2.4}$$

In this part we are employing the fermionic conventions of [14] which we have summarized in appendix C.1. It is also useful to note that the last term in M which is linear in μ is coming from the coupling of fermions to the background self-dual five-form flux. Finally the Wess-Zumino part, after fixing the κ-symmetry as in (2.4), is

$$\begin{aligned}\mathcal{L}_{WZ} \;=\; & -\epsilon^{\hat{\theta}\hat{\mu}\hat{\nu}\hat{\rho}}\partial_{\hat{\theta}}X^+\left[\partial_{\hat{\mu}}X^I\partial_{\hat{\nu}}X^I\bar{\psi}\gamma^{-IJ}\partial_{\hat{\rho}}\psi+\frac{1}{2}F_{\hat{\mu}\hat{\nu}}(\psi\bar{\gamma}^-\partial_{\hat{\rho}}\psi-\bar{\psi}\bar{\gamma}^-\partial_{\hat{\rho}}\bar{\psi})\right] \\ & +\frac{\mu}{6}\epsilon^{\hat{\theta}\hat{\mu}\hat{\nu}\hat{\rho}}\partial_{\hat{\theta}}X^+\left[\epsilon^{ijkl}X^i\partial_{\hat{\mu}}X^j\partial_{\hat{\nu}}X^k\partial_{\hat{\rho}}X^l+\epsilon^{abcd}X^a\partial_{\hat{\mu}}X^b\partial_{\hat{\nu}}X^c\partial_{\hat{\rho}}X^d\right].\end{aligned} \tag{2.5}$$

One can check that the Born-Infeld part and the Wess-Zumino part of the action are individually supersymmetric.

For the later use we separate the symmetric and anti-symmetric parts of the matrix under the square-root:

$$N_{\hat{\mu}\hat{\nu}} \equiv (g_{\hat{\mu}\hat{\nu}}+M_{\hat{\mu}\hat{\nu}})+F_{\hat{\mu}\hat{\nu}} \tag{2.6}$$

and denote its inverse matrix by $N^{\hat{\mu}\hat{\nu}}$. The symmetric and anti-symmetric parts of $N^{\hat{\mu}\hat{\nu}}$ respectively denoted by $G^{\hat{\mu}\hat{\nu}}$ and $\theta^{\hat{\mu}\hat{\nu}}$ have the interpretation of (supersymmetric) *open string metric* and the *noncommutativity parameter* [15]. In what follows we denote the inverse of open string metric by $G_{\hat{\mu}\hat{\nu}}$.

2.2 Fixing the light-cone gauge, the light-cone Hamiltonian

The supersymmetric brane action enjoys three class of local gauge symmetries, the area preserving diffeomorphism (APD) invariance on the worldvolume, its fermionic counterpart the κ-symmetry and the U(1) gauge symmetry. In the light-cone gauge we fix a part of the APD's, those which mix worldvolume time and spatial coordinates while the spatial APD's are still un-fixed. We fix the fermionic κ-symmetry completely by throwing away half of the un-physical original 32 fermions. This latter we have done by imposing (2.4). In the terminology of constrained systems, these gauge fixing conditions are primary constraints and one should make sure the consistency of these constraints, which in turn lead to a set of secondary constraints. This work for the supersymmetric D-brane action, in a general (not necessarily light-cone gauge), but in the flat space background has been performed in [16, 17].

To fix the light-cone gauge we separate the space and time indices on the brane worldvolume as $\sigma^{\hat{\mu}}=(\tau=\sigma^0,\sigma^r)$, $r=1,2,3$ the space indices. The light-cone gauge is fixed by choosing

$$X^+=\tau\ . \tag{2.7}$$

To ensure that the above solution of X^+ is maintained by the dynamics we use the time-space mixing part of the APD's and set the time-space components of the open string metric equal to zero [1], i.e.

$$N^{0r}+N^{r0}\equiv G^{0r}=G_{0r}=\big((g+M)-F(g+M)^{-1}F\big)_{0r}=0. \tag{2.8}$$

[1] This equation parallels the "level-matching condition" in string theory. One way to see this is to write the DBI action in the "Polyakov form" by introducing an auxiliary worldvolume metric, $h_{\hat{\mu}\hat{\nu}}$ (see appendix A). In this language the light-cone gauge fixing amounts to setting $h_{0r}=0$. In order this choice

The above is the generalization of level matching condition for strings to the D3-brane case. It is notable that for the plane-wave case, (2.8) is independent of μ, i.e. it has the same form as in the flat space background. The "quantum version" of this fact has also been manifested in the Gauss law of the tiny graviton matrix theory [18] as well as in the analysis of [19] in which the matrix theory formulation of type IIB string theory is obtained from non-BPS D0-branes. The above can be used to identify $\partial_r X^-$ in terms of other dynamical variables and hence using (2.8) both $X^\pm$ are completely removed from the light-cone dynamics.

In the light-cone gauge, $M_{rs} = 0$ while M_{0r} and M_{00} are non-zero. Moreover, in the light-cone gauge the term in the WZ action which is proportional to the gauge field F becomes a total derivative and hence can be dropped.

Next we note that in the plane-wave background, X^- and X^+ are cyclic variables and hence the corresponding conjugate momenta respectively

$$p^+ = \frac{\partial\mathcal{L}}{\partial(\partial_\tau X^-)} = \frac{1}{g_s}\sqrt{-detN}N^{00}, \qquad H_{lc} \equiv P^- = \frac{\partial\mathcal{L}}{\partial(\partial_\tau X^+)}, \tag{2.9}$$

(H_{lc} is the light-cone Hamiltonian density) are constants of motion.

Using properties of the determinant and some matrix identities, one can eliminate the $\partial_\tau X^-$ dependence in the light-cone Hamiltonian [11] and after some algebraic manipulations (some of which have been gathered in the appendix B) the light-cone Hamiltonian is obtained to be

$$\begin{aligned}\mathcal{H} = \int d^3\sigma\Big\{ &\frac{(P^I)^2}{2p^+} + \frac{(P^I_E)^2}{2p^+} + \frac{1}{2}\mu^2 p^+ (X^I)^2 + \frac{1}{2\cdot 3! p^+ g_s^2}\{X^I, X^J, X^K\}^2 + \frac{1}{2p^+ g_s^2}(B^I)^2 \\ &+ \frac{\mu}{6g_s}\Big(\epsilon^{ijkl}X^i\{X^j, X^k, X^l\} + \epsilon^{abcd}X^a\{X^b, X^c, X^d\}\Big) \\ &+ \mu\psi^{\dagger\alpha\beta}\psi_{\alpha\beta} + \frac{2}{p^+ g_s}\Big(\psi^{\dagger\alpha\beta}(\sigma^{ij})_\alpha^{\ \delta}\{X^i, X^j, \psi_{\delta\beta}\} + \psi^{\dagger\alpha\beta}(\sigma^{ab})_\alpha^{\ \delta}\{X^a, X^b, \psi_{\delta\beta}\}\Big) \\ &+ \mu\psi^{\dagger\dot\alpha\dot\beta}\psi_{\dot\alpha\dot\beta} + \frac{2}{p^+ g_s}\Big(\psi^{\dagger\dot\alpha\dot\beta}(\sigma^{ij})_{\dot\alpha}^{\ \dot\delta}\{X^i, X^j, \psi_{\dot\delta\dot\beta}\} + \psi^{\dagger\dot\alpha\dot\beta}(\sigma^{ab})_{\dot\alpha}^{\ \dot\delta}\{X^a, X^b, \psi_{\dot\delta\dot\beta}\}\Big)\Big\}\end{aligned} \tag{2.10}$$

In the above we have used SO(4)×SO(4) representation for the fermions (see appendix C.2), P^I and P^I_E are respectively momenta conjugate to X^I and the gauge field A_r times $\partial_r X^I$, (B.4), (B.5) and (B.14), and

$$B^I = B^r\partial_r X^I = -\frac{1}{\sqrt{2}}\epsilon^{rsp}F_{sp}\partial_r X^I \ . \tag{2.11}$$

Finally the brackets are Nambu 3-brackets defined as (e.g. see [18])

$$\{F, G, K\} = \epsilon^{rps}\partial_r F\partial_s G\partial_s K. \tag{2.12}$$

---

to be respected by the dynamics one should make sure that the equation of motion for h_{0r} is satisfied, that is we demand

$$\frac{\delta\mathcal{L}}{\delta h_{0r}} = \frac{\partial\mathcal{L}}{\partial h_{0r}} = 0,$$

(or impose it as a constraint). From the "Polyakov form" of DBI action (A.3) it is immediately seen that $\frac{\partial\mathcal{L}}{\partial h_{0r}} = G^{0r}$.

The above Hamiltonian should be supplemented by the secondary constraint coming from the U(1) gauge symmetry:

$$\partial_r P_E^r = 0. \tag{2.13}$$

Noting the results of [17] it can be shown that fixing the light-cone gauge by imposing (2.4) and (2.7), leads to no further secondary constraints. However, one should still make sure that the physical configurations of the above Hamiltonian is satisfying (2.8) which can be simplified to

$$p^+ \partial_r X^- = P^I \partial_r X^I + \bar{\psi}\bar{\gamma}^- \partial_r \psi + \psi \bar{\gamma}^- \partial_r \bar{\psi} + F_{rs} P_E^s. \tag{2.14}$$

The light-cone Hamiltonian is invariant under local three dimensional APD's and also the U(1) gauge symmetry (which can be fixed in any gauge, the light-cone gauge or otherwise). It has also global symmetries, such as $psu(2|2) \times psu(2|2) \times u(1)_H$ superalgebra, which will be made explicit in the next subsection, the $\mathbb{Z}_2$ symmetry which exchanges X^i and X^a directions (or identically exchanges the two $psu(2|2)$ factors of the superalgebra) and the electric-magnetic duality

$$P_E^I \longleftrightarrow \frac{B^I}{g_s}. \tag{2.15}$$

2.3 The light-cone supersymmetry algebra

As mentioned the light-cone Hamiltonian is invariant under the dynamical part of the plane-wave superalgebra which is $psu(2|2) \times psu(2|2) \times u(1)_H$. It happens that the relevant superalgebra to our case is in fact the "extended" $psu(2|2) \times psu(2|2) \times u(1)_H$ superalgebra [20]:

- *The fermionic anti-commutators*

$$\begin{aligned}
\{Q_{\dot\alpha\beta}, Q^{\dagger\dot\rho\lambda}\} &= \delta_{\dot\alpha}^{\dot\rho}\delta_\beta^\lambda \mathcal{H} + \frac{\mu}{2}(i\sigma^{ij})_{\dot\alpha}^{\dot\rho}\delta_\beta^\lambda \mathcal{J}_{ij} + \frac{\mu}{2}\delta_{\dot\alpha}^{\dot\rho}(i\sigma^{ab})_\beta^\lambda \mathcal{J}_{ab} + (i\sigma^{ij})_{\dot\alpha}^{\dot\rho}(i\sigma^{ab})_\beta^\lambda \mathcal{R}_{ijab} \\
\{Q_{\alpha\dot\beta}, Q^{\dagger\rho\dot\lambda}\} &= \delta_\alpha^\rho\delta_{\dot\beta}^{\dot\lambda} \mathcal{H} + \frac{\mu}{2}(i\sigma^{ij})_\alpha^\rho\delta_{\dot\beta}^{\dot\lambda} \mathcal{J}_{ij} + \frac{\mu}{2}\delta_\alpha^\rho(i\sigma^{ab})_{\dot\beta}^{\dot\lambda} \mathcal{J}_{ab} + (i\sigma^{ij})_\alpha^\rho(i\sigma^{ab})_{\dot\beta}^{\dot\lambda} \mathcal{R}_{ijab}
\end{aligned} \tag{2.16}$$

  where $\mathcal{H}$ is the light-cone Hamiltonian and $\mathcal{J}_{ij},\ \mathcal{J}_{ab}$ are generators of the two SO(4)'s.

$$\begin{aligned}
\{Q_{\dot\alpha\beta}, Q_{\dot\rho\lambda}\} &= \mathcal{Z}\epsilon_{\dot\alpha\dot\rho}\epsilon_{\beta\lambda} + \mathcal{Z}_{ijab}(i\sigma^{ij})_{\dot\alpha\dot\rho}(i\sigma^{ab})_{\beta\lambda} \\
\{Q_{\alpha\dot\beta}, Q_{\rho\dot\lambda}\} &= \mathcal{Z}\epsilon_{\alpha\rho}\epsilon_{\dot\beta\dot\lambda} + \mathcal{Z}_{ijab}(i\sigma^{ij})_{\alpha\rho}(i\sigma^{ab})_{\dot\beta\dot\lambda}
\end{aligned} \tag{2.17}$$

- *The fermionic-bosonic commutators*

$$[\mathcal{J}_{ij}, Q_{\alpha\dot\beta}] = \frac{1}{2}(i\sigma^{ij})_\alpha^\rho Q_{\rho\dot\beta}, \qquad [\mathcal{J}_{ij}, Q_{\dot\alpha\beta}] = \frac{1}{2}(i\sigma^{ij})_{\dot\alpha}^{\dot\rho} Q_{\dot\rho\beta} \tag{2.18}$$

$$[\mathcal{J}_{ab}, Q_{\alpha\dot\beta}] = \frac{1}{2}(i\sigma^{ab})_{\dot\beta}^{\dot\rho} Q_{\alpha\dot\rho}, \qquad [\mathcal{J}_{ab}, Q_{\dot\alpha\beta}] = \frac{1}{2}(i\sigma^{ab})_\beta^\rho Q_{\dot\alpha\rho}$$

$$[\mathcal{H}, Q_{\alpha\dot\beta}] = 0, \qquad [\mathcal{H}, Q_{\dot\alpha\beta}] = 0 \tag{2.19}$$

  Note that extensions $\mathcal{R}_{ijab},\ \mathcal{Z}_{ijab}$ are not central because they do not commute with $\mathcal{J}$'s.

It is straightforward, but involves lengthy computations, to show that this superalgebra has an explicit realization in terms of the D3-brane fields as:

$$
\begin{aligned}
Q_{\dot\alpha\beta} = \frac{1}{\sqrt{2p^+}}\int d^3\sigma\Big[&(P^i - i\tilde X^i)(\sigma^i)_{\dot\alpha}{}^{\rho}\psi_{\rho\beta} + \left(\frac{B^i}{g_s} + iP^i_E\right)(\sigma^i)^{\rho}{}_{\dot\alpha}\psi^\dagger_{\rho\beta} \\
&+ (P^a - i\tilde X^a)(\sigma^i)_{\beta}{}^{\dot\rho}\psi_{\dot\alpha\dot\rho} + \left(\frac{B^a}{g_s} + iP^a_E\right)(\sigma^a)^{\dot\rho}{}_{\beta}\psi^\dagger_{\dot\alpha\dot\rho} \\
&- \frac{1}{2g_s}\big(\{X^i,X^a,X^b\}(\sigma^i)_{\dot\alpha}{}^{\rho}(i\sigma^{ab})_{\beta}{}^{\theta}\psi_{\rho\theta} + \{X^i,X^j,X^a\}(\sigma^a)_{\beta}{}^{\dot\theta}(i\sigma^{ij})_{\dot\alpha}{}^{\dot\rho}\psi_{\dot\rho\dot\theta}\big)\Big]
\end{aligned}
\tag{2.20}
$$

where

$$
\begin{aligned}
\tilde X^i &= \mu p^+ X^i + \frac{1}{3!g_s}\epsilon^{ijkl}\{X^j,X^k,X^l\} \\
\tilde X^a &= \mu p^+ X^a + \frac{1}{3!g_s}\epsilon^{abcd}\{X^b,X^c,X^d\},
\end{aligned}
\tag{2.21}
$$

and similarly for the $Q_{\alpha\dot\beta}$. The Hamiltonian is of course given by (2.10) and

$$
\begin{aligned}
\mathcal{J}_{ij} &= \int d^3\sigma\Big[X^iP^j - X^jP^i + \frac{1}{\mu p^+ g_s^2}(P^i_E B^j - P^j_E B^i) - 2\psi^{\dagger\alpha\beta}(i\sigma^{ij})^{\rho}_{\alpha}\psi_{\rho\beta} + 2\psi^{\dagger\dot\alpha\dot\beta}(i\sigma^{ij})^{\dot\rho}_{\dot\alpha}\psi_{\dot\rho\dot\beta}\Big] \\
\mathcal{J}_{ab} &= \int d^3\sigma\Big[X^aP^b - X^bP^a + \frac{1}{\mu p^+ g_s^2}(P^a_E B^b - P^b_E B^a) - 2\psi^{\dagger\alpha\beta}(i\sigma^{ab})^{\rho}_{\beta}\psi_{\alpha\rho} + 2\psi^{\dagger\dot\alpha\dot\beta}(i\sigma^{ab})^{\dot\rho}_{\dot\beta}\psi_{\dot\alpha\dot\rho}\Big]
\end{aligned}
\tag{2.22}
$$

and the extensions are obtained to be

$$
\begin{aligned}
\mathcal{Z} &= \frac{1}{p^+}\int d^3\sigma\left[(P^I - i\tilde X^I)\left(\frac{B^I}{g_s} + iP^I_E\right)\right] \\
\mathcal{Z}_{ijab} &= -\frac{i}{4p^+ g_s}\int d^3\sigma\left[\left(\frac{B^i}{g_s} + iP^i_E\right)\{X^j,X^a,X^b\} + i,j \leftrightarrow\ a,b\right] \\
\mathcal{R}_{ijab} &= \frac{\mu}{g_s}\int d^3\sigma\{X^i,X^j,X^a\}X^b
\end{aligned}
\tag{2.23}
$$

To verify the above commutations relations we have employed the basic Poisson brackets:

$$
[X^I(\sigma), P^J(\sigma')] = i\delta^{IJ}\delta(\sigma-\sigma') \tag{2.24}
$$

$$
\begin{aligned}
[A_r(\sigma), P^s_E(\sigma')] &= i\delta^s_r\delta(\sigma-\sigma') \\
\{\psi_{\alpha\beta}(\sigma), \psi^{\dagger\rho\lambda}(\sigma')\} &= \delta^\rho_\alpha\delta^\beta_\lambda\delta(\sigma-\sigma') \\
\{\psi_{\alpha\beta}(\sigma), \psi^\dagger_{\rho\lambda}(\sigma')\} &= \epsilon_{\alpha\rho}\epsilon_{\beta\lambda}\delta(\sigma-\sigma')\ ,
\end{aligned}
\tag{2.25}
$$

where (2.25) is subject to $\partial_r P^r_E = 0$. We choose to fix the U(1) symmetry in the Coulomb gauge $A_0 = 0$.

2.4 1/2 BPS configurations

From the superalgebra given in the previous section it is clear that for the 1/2 BPS configurations of our model, those which preserve 16 (that is all of the) supercharges, the right-hand-side of the fermionic anticommutators should vanish. That is, for 1/2 BPS configurations we must have

$$\mathcal{H} = 0, \quad \mathcal{J}_{ij} = \mathcal{J}_{ab} = 0, \qquad \mathcal{Z} = 0, \quad \mathcal{R}_{ijab} = \mathcal{Z}_{ijab} = 0. \tag{2.26}$$

The above is only possible if we turn off fermions, $P^I = P^I_E = 0$, $B^I = 0$ and $\tilde{X}^i = 0,\ X^a = 0$ or $\tilde{X}^a = 0,\ X^i = 0$ ($\tilde{X}^i,\ \tilde{X}^a$ are defined in (2.21)). These two choices are related by the $i \leftrightarrow a$ exchange $\mathbb{Z}_2$ symmetry, therefore we only consider the $\tilde{X}^i = 0,\ X^a = 0$ case, that is [11]

$$\{X^i, X^j, X^k\} = -\mu p^+ g_s\, \epsilon^{ijkl} X_l\ . \tag{2.27}$$

Eq.(2.27) has two SO(4) invariant solutions:

(i) $X^i = 0$ which specifies a zero size point like spherical 3-brane and,

(ii) A three sphere of radius R [11]

$$R^2 = \mu p^+ g_s\,, \tag{2.28}$$

(if we recover α''s that is $R^2 = 2\pi\mu p^+ \alpha' g_s$).[2]

To verify this it is enough to take

$$\begin{aligned} X^1 &= R\sin\psi\sin\theta\cos\phi, \quad & X^2 &= R\sin\psi\sin\theta\sin\phi \\ X^3 &= R\sin\psi\cos\theta, & X^4 &= R\cos\psi \end{aligned} \tag{2.29}$$

and recall that in the above coordinates

$$\{F, G, K\} = \frac{1}{\sin^2\psi\sin\theta}\epsilon^{rps}\partial_r F\ \partial_p G\ \partial_s K\,,$$

where now ϵ^{rps} the totally antisymmetry Levi-Civita tensor and takes only $0, \pm 1$ values. This spherical solutions are the giant gravitons on the plane-wave background.

The fact that the above solutions are 1/2 BPS can also be seen directly from the supercharges and that the supersymmetric variations of fermions

$$\begin{aligned} \delta\psi_{\rho\lambda} &= i\{\epsilon^{\dagger\alpha\dot{\beta}} Q_{\alpha\dot{\beta}} + \epsilon_{\alpha\dot{\beta}} Q^{\dagger\alpha\dot{\beta}} + \epsilon^{\dagger\dot{\alpha}\beta} Q_{\dot{\alpha}\beta} + \epsilon_{\dot{\alpha}\beta} Q^{\dagger\dot{\alpha}\beta}\ , \psi_{\rho\lambda}\} \\ \delta\psi_{\dot{\rho}\dot{\lambda}} &= i\{\epsilon^{\dagger\alpha\dot{\beta}} Q_{\alpha\dot{\beta}} + \epsilon_{\alpha\dot{\beta}} Q^{\dagger\alpha\dot{\beta}} + \epsilon^{\dagger\dot{\alpha}\beta} Q_{\dot{\alpha}\beta} + \epsilon_{\dot{\alpha}\beta} Q^{\dagger\dot{\alpha}\beta}\ , \psi_{\dot{\rho}\dot{\lambda}}\} \end{aligned} \tag{2.30}$$

vanish for all sixteen possible supersymmetry transformation parameters $\epsilon_{\alpha\dot{\beta}}$, $\epsilon_{\dot{\alpha}\beta}$, once we plug in the above three spherical solutions.

[2] It is worth noting that fixing the light-cone gauge from the viewpoint of the above 1/2 BPS configurations corresponds to going to the rest frame of these objects. From the background plane-wave point of view, these are objects following the light-like geodesic $\frac{\partial}{\partial X^-}$ with the momentum p^+ along the light-like trajectory. This also justifies the name giant *graviton*.

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3. BPS configurations with static electric fields

In this section we study BPS configurations involving given electromagnetic fields. The case of our interest is mainly the static electromagnetic field, however, we will also briefly discuss the BPS electromagnetic waves. These configurations can be classified by the amount of supersymmetry they preserve. A class of less BPS configurations can be understood as deformations of 1/2 BPS states discussed in the previous section. For a general (less) BPS state the supersymmetry transformations may vanish for some specific choices for the supersymmetry transformation parameters. The number of supersymmetries preserved is then the number of independent *real* $\epsilon_{\alpha\dot\beta}$, $\epsilon_{\dot\alpha\beta}$'s which satisfy $\delta\psi_{\rho\lambda}=0\,,\delta\psi_{\dot\rho\dot\lambda}=0$ equation for that specific configuration.

For example, consider the spherical 1/2 BPS giant graviton configuration, but now turn on electric and magnetic fields, such that

$$P_E^2=\frac{1}{g_s}B^1, \tag{3.1}$$

and all the other components are zero. The above describes a 1/4 BPS state of a photon, for which $\mathcal{H}=\mu\mathcal{J}_{12}=\frac{1}{p^+}P_E^2$ [11]. (Note that the time dependence of the gauge field is given by its equations of motion which for this case is basically the same as Maxwell equations on $R\times S^3$ and we do not present them here explicitly.) One can of course construct less BPS electromagnetic waves (photons) which all propagate on the spherical three brane by superposing various photon states propagating in different directions [21].

In the rest of this section we only focus on the static electromagnetic fields. Requiring the configurations involving given static electric or magnetic fields to be BPS, as we will see, forces us to deform the shape of the three sphere.

Here we only study static configurations, that is we set $P^I=0$, which are deformations of three sphere giants in the X^i direction, that is we set $X^a=0$ and $P_E^a=B^a=0$, and of course turn the fermions off. For this specific class the BPS condition simplifies to

$$\delta\psi_{\rho\lambda} = \left[i\tilde{X}^i\epsilon_{\dot\alpha\lambda}-\left(\frac{B^i}{g_s}+iP_E^i\right)\epsilon^\dagger_{\dot\alpha\lambda}\right](\sigma^i)^{\dot\alpha}{}_{\rho}=0 \tag{3.2}$$

$$\delta\psi_{\dot\rho\dot\lambda} = \left[i\tilde{X}^i\epsilon_{\alpha\dot\lambda}-\left(\frac{B^i}{g_s}+iP_E^i\right)\epsilon^\dagger_{\alpha\dot\lambda}\right](\sigma^i)_{\dot\rho}{}^{\alpha}=0 \tag{3.3}$$

For these configurations it is evident that $\mathcal{J}_{ab}=0$ and $\mathcal{R}_{ijab}=\mathcal{Z}_{ijab}=0$. The only non-vanishing bosonic generators can hence be $\mathcal{J}_{ij}$, $\mathcal{H}$ and $\mathcal{Z}$ (*cf.* (2.22) and (2.23)). $\mathcal{H}$ is positive definite and only vanishes for the three sphere giants and for all these configurations $\mathcal{H}\neq 0$. As we will show, for 1/4 BPS configurations satisfying (3.2) and (3.3) $\mathcal{J}_{ij}$ also vanishes and for all of the 1/4 BPS configurations the BPS condition is realized as $\mathcal{H}=\pm\mathcal{Z}$. In this sense we consider new class of BPS solutions to the three brane giant graviton theory which has not been studied in the literature before. (In the literature mainly the configurations with non-vanishing $\mathcal{J}_{ij}$, $\mathcal{J}_{ab}$, $\mathcal{R}_{ijab}$ have been considered e.g. see [11, 18, 22, 23]).

The BPS equations (3.2) and (3.3) are relating ϵ and $\epsilon^\dagger$ and therefore they are only satisfied if $\tilde{X}^i$ and Π^i,

$$\Pi^i\equiv\frac{B^i}{g_s}+iP_E^i, \tag{3.4}$$

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are related in a specific way. For 1/4 BPS configurations this happens if and only if

$$\Pi^i \bar{\Pi}^j = \Pi^j \bar{\Pi}^i \tag{3.5}$$
$$\tilde{X}^i \tilde{X}^i = \Pi^i \bar{\Pi}^i. \tag{3.6}$$

(3.5) can also be written as

$$P_E^i B^j = P_E^j B^i \tag{3.7}$$

and therefore for 1/4 BPS configurations $\mathcal{J}_{ij} = 0$. Note that once (3.5) are fulfilled (3.2) and (3.3) become identical. (A more detailed analysis leading to the above equations may be found in appendix D.) Eq.(3.7) is satisfied if either P_E^i or B^i is vanishing or when P_E is parallel to B when both are non-zero. In this paper we only consider the case with non-zero P_E^i, $B^i = 0$. The case with vanishing P_E^i and non-zero B can be obtained from the former using the electric-magnetic duality (2.15).

For the pure electric case ($B^i = 0$), (3.5) simplifies to

$$\tilde{X}^i \tilde{X}^i = P_E^i P_E^i. \tag{3.8}$$

Since two SO(4) vectors of the same norm are always related by an SO(4) rotation,

$$\tilde{X}^i = R^{ij} P_E^j, \qquad R^{ij} R^{kj} = \delta^{ik}. \tag{3.9}$$

Before studying specific solutions, we also discuss the BPS "perfect square trick". For the cases with only non-zero $\tilde{X}^i$ and P_E^i the Hamiltonian takes the simple form

$$\begin{aligned} \mathcal{H} &= \frac{1}{2p^+} \int d^3\sigma \; [(\tilde{X}^i)^2 + (P_E^i)^2] \\ &= \frac{1}{2p^+} \int d^3\sigma \; [(\tilde{X}^i \pm R^{ij} P_E^j)^2 \mp 2\tilde{X}^i R^{ij} P_E^j] \end{aligned} \tag{3.10}$$

where $R_{ij} R_{kj} = \delta_{ik}$ is an SO(4) rotation. The usual BPS arguments then tells us that $\mathcal{H}$ is minimized when $\tilde{X}^i = R^{ij} P_E^j$.

In the rest of this section we study solutions to (3.9) for given specific static electric fields. We analyze two class of solutions. In section 3.1 we study cases which are of the form of giant three branes deformed as a result of the electric field. In section 3.2 we study cases where we have string type configurations. These configurations may be thought as extremely deformed three branes which effectively behave like fundamental strings or equivalently as deformations about $X = 0$ vacuum.

3.1 Giant-like configurations

In this section we turn on electric fields on the giant graviton and study its shape deformation induced by the field. We consider two cases, first the case where the electric field is sourced by two equally charged but opposite point charges placed on the North and South poles of the three sphere, and second we study the constant electric field on the brane.

3.1.1 BIGGons, BIons on the giant gravitons

Consider the electric fields sourced by point charges on the three sphere giants. Since the three sphere is compact we cannot place non-zero net charge on it and hence the simplest possibility is an electric dipole composed of a plus and a minus charge put on the South and North poles of the three sphere giant. To make this configuration BPS we need to turn on X^i in a particular way, dictated by the BPS equations (3.9). This generalizes the BIons to the giant gravitons. This problem was first considered in [11], where it was only analyzed in the Hamiltonian which is expanded up to quadratic order in the fields. Here we intend to make the full Born-Infeld analysis, to all orders in fields. Nonetheless as we will show the amount of supersymmetries and bosonic symmetries of the system remains the same compared to the second order analysis of the Hedgehog case.

Let us start by solving for the electric field:

$$\vec{\nabla}.\vec{E} = \frac{1}{\sin^2\psi}\partial_\psi\big(\sin^2\psi E\big) = \frac{Q}{\sin^2\psi}\big(\delta(\psi)-\delta(\pi-\psi)\big)\delta(\cos\theta)\delta(\phi) \tag{3.11}$$

yielding

$$E^\psi = \frac{Q}{\sin^2\psi}\ . \tag{3.12}$$

The above electric field keeps the $\mathrm{SU}(2)_D$ (which acts on θ,ϕ directions) and X^i should also be turned on keeping the same $\mathrm{SU}(2)_D$, i.e.

$$\begin{aligned} X^1 &= RS(\psi)\sin\theta\cos\phi \\ X^2 &= RS(\psi)\sin\theta\sin\phi \\ X^3 &= RS(\psi)\cos\theta \\ X^4 &= RC(\psi) \end{aligned} \tag{3.13}$$

The BPS equation (3.9) is then written as

$$\begin{aligned} \sin^2\psi S + S^2C' &= \sigma\lambda(S'\cos\alpha + C'\sin\alpha) \\ \sin^2\psi C - S^2S' &= \sigma\lambda(C'\cos\alpha - S'\sin\alpha) \end{aligned} \tag{3.14}$$

where we have used the definition of P^i_E, $P^i_E = E^\psi\partial_\psi X^i$, α is the rotation angle relating $\tilde{X}^i$ and P^i_E and σ is just a plus or minus sign. (Although σ could be absorbed in the definition of rotation angle α, it turns out to be more convenient to keep σ.) Finally, $S' = \frac{dS}{d\psi}$, $C' = \frac{dC}{d\psi}$ and $\lambda = \frac{Q}{\mu}$. Without loss of generality we take λ to be positive.

It is readily seen that both side of above equations under the parity transformation $\theta\,,\ \psi \to \pi-\theta\,,\ \pi-\psi$ and $\phi\to\pi+\phi$ behave in the same way if under parity $\alpha\to\pi-\alpha$ and

$$S(\psi) = S(\pi-\psi), \qquad C(\psi) = -C(\pi-\psi).$$

The BPS equations take a simpler form in terms of "polar coordinate variable"

$$\begin{aligned} S &= r(\psi)\sin\chi(\psi) \\ C &= r(\psi)\cos\chi(\psi). \end{aligned} \tag{3.15}$$

For $Q = 0$ case it is evident that $r(\psi) = 1$ and $\chi = \psi$ is a solution to (3.14). Deviation of χ from ψ then comes from the charges we have in the system. Under parity χ should also behave the same as ψ, i.e.

$$\chi(\psi) = \pi - \chi(\pi - \psi)$$

and $r(\psi) = r(\pi - \psi)$.

In the BPS equations α is an arbitrary angle but should transform suitably under parity. With the choice

$$\alpha = \chi$$

the BPS equations, which are non-linear coupled first order differential equations for r and χ take a simple form and could be solved. With this choice straightforward algebra leads to

$$r = 1 - \frac{\sigma\lambda}{S} \tag{3.16}$$

where we have used the initial condition that for $\lambda = 0$, $r = 1$. Using the above one can eliminate r to obtain the equation for S (or χ)

$$\sigma' \sin^2\psi = \left(\frac{S^5 - 3\sigma\lambda S^4 + \lambda^2 S - \sigma\lambda^3}{S(S - \sigma\lambda)\sqrt{S^2 + \lambda^2 - 2\sigma\lambda S - S^4}}\right) S' \tag{3.17}$$

where σ' is +1 (-1) for $\psi < \pi/2$ ($\psi > \pi/2$). Although the shape of the giant is completely determined by (3.16), equation (3.17) is needed to find the range over which the variable S is varying. Recalling that $\sin\chi = r/S$ from (3.16) we learn that

$$0 \leq \sin\chi = \frac{S^2}{S - \sigma\lambda} \leq 1 \tag{3.18}$$

and hence

$$\frac{1}{2}(1 - \sqrt{1 - 4\sigma\lambda}) \leq S \leq \frac{1}{2}(1 + \sqrt{1 - 4\sigma\lambda}). \tag{3.19}$$

Let us now consider the $\sigma = +$ and $\sigma = -$ cases separately:

- $\sigma = -$, *the outward spike*: for this case (3.16) and (3.19) read as

$$r = 1 + \frac{\lambda}{S}, \qquad 0 \leq S \leq \frac{1}{2}(1 + \sqrt{1 + 4\lambda}), \tag{3.20}$$

  and hence $r > 1$. If we ignore (3.17), (3.20) describes a three sphere with two spikes *going off to infinity* coming out of the two poles. The lowest value S can take S_0, is however, further restricted by (3.17) and S can never become zero. Hence,the spikes are cut off and do not go to infinity. This is in fact the main qualitative difference of the full Born-Infeld analysis compared to the case of [11].

  To see this let us suppose that S can become arbitrarily small and study (3.17) in the $S \to 0$ region which while keeping λ fixed, necessarily happens when $\psi \to 0$. In this limit equation (3.17) implies that

$$S \sim k e^{-\frac{1}{3!\lambda}\psi^3}, \tag{3.21}$$

where k is an integration constant. This is a contradiction, as for small ψ and finite k, S does not approach zero. Therefore, S cannot become smaller than some S_0 where the spike is cut off. S_0 is a complicated function of λ which in principle can be computed integrating (3.17). However, from (3.17), one can deduce that when $\lambda \to 0$, $S_0(\lambda)$ also tends to zero.

In sum, the shape of the brane is given by

$$r = 1 + \frac{\lambda}{S}, \qquad S_0(\lambda) \leq S \leq \frac{1}{2}(1 + \sqrt{1 + 4\lambda}). \tag{3.22}$$

This function is depicted in figure 1.

- $\sigma = +$, *the inward spike*: for this case (3.16) and (3.19) read as

$$r = 1 - \frac{\lambda}{S}, \qquad \frac{1}{2}(1 - \sqrt{1 - 4\lambda}) \leq S \leq \frac{1}{2}(1 + \sqrt{1 - 4\lambda}), \tag{3.23}$$

  and hence always $r < 1$. Since r cannot take negative values $S \geq \lambda$, furthermore in order (3.18) to hold we must have $\lambda \leq 1/4$. For $\lambda > 1/4$ there is no BPS inward spike solution. If $\lambda \leq 1/4$ then $\frac{1}{2}(1 - \sqrt{1 - 4\lambda}) \leq \lambda$ and hence S can take all the values in the range given in (3.23). For the maximal and minimal values of S, it is easily seen that $r_{\min} = S_{\min},\ r_{\max} = S_{\max}$ and that

$$\frac{1}{2}(1 - \sqrt{1 - 4\lambda}) \leq r,\ S \leq \frac{1}{2}(1 + \sqrt{1 - 4\lambda}).$$

  Therefore, the size of the throat in the spike is $S_{\min}$. Moreover, at $S_{\min}$ and $S_{\max}$ the slope of the curve is infinite (i.e. the tangent is parallel to vertical axes). The inward spike has the topology of $S^2 \times S^1$ and has been depicted in figure 2.

  For $\lambda = 1/4$, the only value that S or r can take is $r = S = 1/2$ and the giant becomes a two-sphere of radius 1/2. If λ is larger than 1/4 the solution is not smoothly connected to the spherical three brane giant.

It is instructive to compare our analysis to the case of giant Hedgehog [11] which is obtained from our equations if we first take $\lambda \ll 1$ limit. In this limit one can drop the non-linear terms in (3.14) and $\chi = \alpha \simeq \psi$. In this case the spikes go off to infinity. For generic values of ψ (when ψ is not close to 0 or π) the value of the electric field (3.12) as well as $\partial_\psi X^i$ (or S' and C') are small and the quadratic approximation in the Hamiltonian is a good one. Close to $\psi = 0$ or $\psi = \pi$ region, however, we start seeing deviations of Born-Infeld from the quadratic analysis and the Maxwell approximation is not a valid one. As discussed the non-linear terms around $\psi = 0$ become dominant and cap off the spike. This is in contrast to the flat brane case where for both of the second order Maxwell theory and the full Born-Infeld cases the spike is infinite [3, 5].

As a result of the deformation in the shape of the three sphere giant the singularity of the point charges has been removed. To see this let us work out the value of the norm of

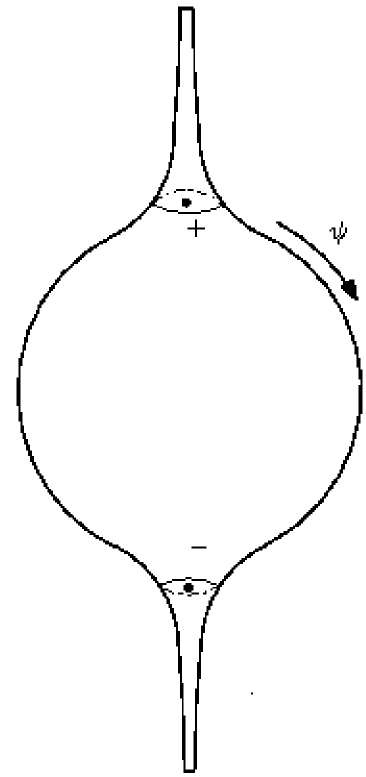

Figure 1: The shape of the outward BIGGons which is obtained for the choice $\sigma = -$ in (3.16). The spike is cut off and does not go to infinity. The minimum value of r is obtained when S takes its maximal value, $r_{\min} = S_{\max} = \frac{1}{2}(1+\sqrt{1+4\lambda})$. The maximum value of r, which is basically (one plus) the height of the spike is $1+\lambda/S_0$. Note that at $S = S_0$ the slope of the spike is large but still finite.

the electric field:

$$\begin{aligned} E^2 = (E^{\psi})^2 \cdot g_{\psi\psi} &= \frac{Q^2 R^2}{\sin^4\psi}(C'^2 + S'^2) \\ &= Q^2 R^2 \frac{(S-\sigma\lambda)^3}{S^4(S^5 - 3\sigma\lambda S^4 + \lambda^2 S - \sigma\lambda^3)}. \end{aligned} \tag{3.24}$$

The factor in the denominator does not vanish for neither of the inward and outward spikes (note that S never becomes zero). The energy density of the solution,

$$\mathcal{H} = p^+ \int d\psi d\theta d\phi \;\; \sin^2\psi \sin\theta \; E^2, \tag{3.25}$$

where E^2 is the expression given in (3.24), is hence finite.

3.1.2 Squashed Giant configuration

As the second example let us consider the case with a given constant electric field on the brane

$$P_E^i = P_E^r \partial_r X^i = P_E^\phi \partial_\phi X^i, \quad P_E^\phi = p^+ E = constant \tag{3.26}$$

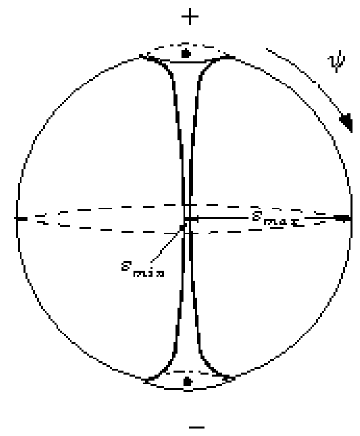

Figure 2: The $\sigma = +$ in (3.16) gives the inward BIGGon. This case is obtained only if $\lambda \leq 1/4$. The minimum and maximum value of the radius r is given by the minimum and maximum values of S, respectively, $\frac{1}{2}(1-\sqrt{1-4\lambda})$ and $\frac{1}{2}(1+\sqrt{1-4\lambda})$.

with this electric field the BPS equation (3.9) takes the form

$$\begin{aligned}\tilde{X}^1 &= \sigma(P_E^1 \cos\alpha + P_E^2 \sin\alpha)\\ \tilde{X}^2 &= \sigma(P_E^2 \cos\alpha - P_E^1 \sin\alpha)\\ \tilde{X}^3 &= 0\\ \tilde{X}^4 &= 0\end{aligned} \tag{3.27}$$

where again σ takes ± 1 values and α is a yet-to-be-fixed angle. In the following we choose E to be positive. Equation (3.27) is solved with

$$\begin{aligned}X^1 &= Ra \sin\psi \sin\theta \cos\phi\\ X^2 &= Ra \sin\psi \sin\theta \sin\phi\\ X^3 &= Rb \sin\psi \cos\theta\\ X^4 &= Rb \cos\psi\end{aligned} \tag{3.28}$$

provided that

$$b^2 = 1 - \sigma\frac{E}{\mu} \ \ , \ \ a^2 = 1 \ \ , \ \ \alpha = \pi/2. \tag{3.29}$$

The above solution describes a brane with a deformed sphere ellipsoidal shape. Shape of the three brane can be easily worked out

$$X_1^2 + X_2^2 + \frac{1}{1-\sigma E/\mu}(X_3^2 + X_4^2) = R^2. \tag{3.30}$$

For $\sigma = -1$ the configuration exist for all values of electric field. For $\sigma = +1$ only for $E < \mu$ we have an ellipsoid, for $E = \mu$ the shape is singular (see section 3.2 for more details) and for $E > \mu$ the brane has a hyperboloid shape. For both of the signs, when we have ellipsoids, there is a 2-sphere cross section. To see this set $X_3 = \sqrt{1-\sigma E/\mu}\, r\sin\gamma$ and $X_4 = \sqrt{1-\sigma E/\mu}\, r\cos\gamma$. Then we recover a 2-sphere of radius R in $r12$-space. This shows the SU(2) isometry of the solution. There is a circular cross section e.g. at $X^3 = X^4 = 0$ with radius R. Therefore our 1/4 BPS configuration keeps SU(2) $\times$ U(1) isometries out of the whole SO(4). Similar ellipsoid branes can also been obtained from a rotating three sphere giants [22]. Total energy for these configuration can be evaluated

$$\mathcal{H} = \int d^3\sigma p^+ E^2 \big((\partial_\phi X^1)^2 + (\partial_\phi X^2)^2\big) = \pi^2 \mu g_s (p^+ E)^2. \tag{3.31}$$

3.2 String-like configurations

In this section we study 1/4 BPS string-like solutions of (2.10) which involve constant background electric field. As discussed (2.10) has two kind of 1/2 BPS configurations, a single giant three sphere of radius $R^2 = \mu p^+ g_s$, or a zero size brane siting at $X^i = X^a = 0$. The string-like solutions of this section can then be understood either as extremely deformed (squashed) three branes or as deformation about the $X = 0$ solution. Here we consider examples of each kind in the background electric field

$$P^i_E = P^r_E \partial_r X^i = P^\phi_E \partial_\phi X^i \ , \ P^\phi_E = p^+ E = constant \tag{3.32}$$

while $X^3 = X^4 = 0$.

For this choice, the BPS equation (3.9) becomes

$$\begin{aligned} R^2 X^1 &= \sigma g_s (P^1_E \cos\alpha + P^2_E \sin\alpha) \\ R^2 X^2 &= \sigma g_s (P^2_E \cos\alpha - P^1_E \sin\alpha) \end{aligned} \tag{3.33}$$

3.2.1 Circular string

In order to find other string-like BPS solution we start with

$$X^1 = Ra\cos\phi \, , \ X^2 = Rb\sin\phi \tag{3.34}$$

(while $\psi = \theta = \frac{\pi}{2}$). The above solves (3.33) if

$$E^2 = \mu^2 \ , \quad a^2 = b^2 \ , \quad \alpha = \pi/2. \tag{3.35}$$

This configuration which is a circular closed string of radius Ra, is a special case of the squashed giant of section 3.2.1 with $\sigma = +$ and $E = \mu$. In this sense this solution is an example of extremely deformed three brane giant. In another viewpoint if we turn off the electric field this solution reduces to the $X = 0$ vacuum. The total energy density of this configuration is $\mathcal{E} = \mu g_s a^2 (p^+ E)^2$.

This configuration is in fact a fundamental string in the X^1, X^2 plane, in the pp-wave background. To see it we consider fluctuations around this solution

$$X^1 = X^1_0 + Y^1 \, , \ X^2 = X^2_0 + Y^2 \tag{3.36}$$

where X_0^i are the the circular string solution given in (3.34). The Hamiltonian for these fluctuations is

$$H = \frac{1}{2p^+}(P_1^2 + P_2^2 + P_3^2 + P_4^2) + \frac{1}{2}\mu^2 p^+(X_3^2 + X_4^2) + \frac{1}{2}\mu^2 p^+(Y_1^2 + Y_2^2) + \frac{\mu^2 p^+}{2}\big((\partial_\phi Y_1)^2 + (\partial_\phi Y_2)^2\big). \tag{3.37}$$

which is the light-cone Hamiltonian for a four dimensional string in the plane-wave background. The tension term, the last term, is coming from the electric field. The tension of this string is $\mu = E$. This string can be thought as an array of tiny electric dipoles which are aligned in the background electric field. In this picture it becomes clear that the tension is proportional to the electric field and that string is only in the plane where the electric field is turned on.

3.2.2 Stretched string

(3.34) solves (3.33) with another value for α:

$$a(\phi)\cos\phi = ce^{\pm\frac{\mu}{E}\phi}\,,\ b(\phi)\sin\phi = de^{\pm\frac{\mu}{E}\phi}\,,\quad \alpha = 0 \tag{3.38}$$

with no restriction on E. Physically for vanishing electric field we should recover the $X = 0$ vacuum, therefore plus sign in the above solution is not acceptable. The above describes a straight string along the

$$X^2 = \frac{d}{c}X^1 \tag{3.39}$$

line with $Rce^{-\frac{2\pi\mu}{E}} \leq X^1 \leq Rc$. The string length is equal to

$$l = R(c^2 + d^2)^{\frac{1}{2}}(1 - e^{-\frac{2\pi\mu}{E}})^{\frac{1}{2}}. \tag{3.40}$$

The maximum possible length is then obtained for $E \to \infty$. The energy density of this configuration is $\mathcal{E} = \mu^2 p^+ l^2$.

This configuration is describing a fundamental string of tension E. To see this one can work out the Hamiltonian for the fluctuation about this solution. Inserting (3.36) into the Hamiltonian we obtain an expression similarly to (3.37) but now the tension is equal to the electric field E. This kind of string, similarly to the previous case, is composed of a set of electric dipoles which are ordered in opposite direction to the electric field E.

3.3 Superalgebra viewpoint

To complete our BPS analysis we also study the BPS configurations we have discussed directly from the superalgebra point of view. As we discussed in general the rotation angle α is not a constant and can in general be a function of ψ, θ or ϕ. To include the effects of this angle into the superalgebra, therefore we need to modify the supercharge densities given in (2.20) to include the rotation R_{ij}. It is straightforward to check that

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anti-commutator of the supercharges

$$\hat{Q}_{\dot{\alpha}\beta} = \frac{1}{\sqrt{2p^+}}\int d^3\sigma\Bigg[(P^i - i\tilde{X}^i)(\sigma^i)_{\dot{\alpha}}{}^{\rho}\psi_{\rho\beta} + \left(\frac{B^i}{g_s} + iP_E^i\right)R^{ij}(\sigma^j)^{\rho}{}_{\dot{\alpha}}\psi^{\dagger}_{\rho\beta} \tag{3.41}$$
$$+(P^a - i\tilde{X}^a)(\sigma^i)_{\beta}{}^{\dot{\rho}}\psi_{\dot{\alpha}\dot{\rho}} + \left(\frac{B^a}{g_s} + iP_E^a\right)R^{ab}(\sigma^b)^{\dot{\rho}}{}_{\beta}\psi^{\dagger}_{\dot{\alpha}\dot{\rho}}$$
$$-\frac{1}{2g_s}\left(\{X^i, X^a, X^b\}(\sigma^i)_{\dot{\alpha}}{}^{\rho}(i\sigma^{ab})_{\beta}{}^{\theta}\psi_{\rho\theta} + \{X^i, X^j, X^a\}(\sigma^a)_{\beta}{}^{\dot{\theta}}(i\sigma^{ij})_{\dot{\alpha}}{}^{\dot{\rho}}\psi_{\dot{\rho}\dot{\theta}}\right)\Bigg]$$

where R_{ij} is defined in (3.9), with its complex conjugate reproduces the light-cone Hamiltonian but with modified $\mathcal{J}_{ij}$, $\mathcal{J}_{ab}$, $\mathcal{R}_{ijab}$. One can also work out anti-commutators of two supercharges to read the central extension $\hat{\mathcal{Z}}$. For the 1/4 BPS configuration of our interest where only $\mathcal{H}$, $\hat{\mathcal{Z}}$ are non-vanishing,

$$\{\hat{Q}_{\dot{\alpha}\beta}, \hat{Q}^{\dagger\ \dot{\rho}\lambda}\} = \mathcal{H}\delta^{\dot{\rho}}_{\dot{\alpha}}\delta^{\lambda}_{\beta}\ , \qquad \{\hat{Q}_{\alpha\dot{\beta}}, \hat{Q}^{\dagger\ \rho\dot{\lambda}}\} = \mathcal{H}\delta^{\rho}_{\alpha}\delta^{\dot{\lambda}}_{\dot{\beta}} \tag{3.42}$$

and

$$\{\hat{Q}_{\dot{\alpha}\beta}, \hat{Q}_{\dot{\rho}\lambda}\} = \hat{\mathcal{Z}}\epsilon_{\dot{\alpha}\dot{\rho}}\epsilon_{\beta\lambda}\ , \qquad \{\hat{Q}_{\alpha\dot{\beta}}, \hat{Q}_{\rho\dot{\lambda}}\} = \hat{\mathcal{Z}}\epsilon_{\alpha\rho}\epsilon_{\dot{\beta}\dot{\lambda}} \tag{3.43}$$

where

$$\hat{\mathcal{Z}} = \frac{1}{p^+}\int d^3\sigma\left[(P^i - i\tilde{X}^i)R^{ij}\left(\frac{B^j}{g_s} + iP_E^j\right) + (P^a - i\tilde{X}^a)R^{ab}\left(\frac{B^b}{g_s} + iP_E^b\right)\right]. \tag{3.44}$$

$\hat{\mathcal{Z}}$ as well as the Hamiltonian $\mathcal{H}$ take different values for each configurations. However, it is readily seen that for all the configurations studied in this section $\mathcal{H} = \pm\hat{\mathcal{Z}}$. To see the BPS nature of these configurations explicitly, we note that the superalgebra produced by the supercharges

$$\tilde{Q}_{\dot{\alpha}\beta} = \frac{1}{\sqrt{2}}\left(\hat{Q}_{\dot{\alpha}\beta} \pm \epsilon_{\dot{\alpha}\dot{\rho}}\epsilon_{\beta\rho}\hat{Q}^{\dagger\dot{\rho}\rho}\right)$$
$$\tilde{Q}_{\alpha\dot{\beta}} = \frac{1}{\sqrt{2}}\left(\hat{Q}_{\alpha\dot{\beta}} \pm \epsilon_{\alpha\rho}\epsilon_{\dot{\beta}\dot{\rho}}\hat{Q}^{\dagger\rho\dot{\rho}}\right) \tag{3.45}$$

is of the form

$$\{\tilde{Q}_{\dot{\alpha}\beta}, \tilde{Q}^{\dagger\dot{\rho}\lambda}\} = \delta_{\dot{\alpha}}{}^{\dot{\rho}}\delta_{\beta}{}^{\lambda}(\mathcal{H} \pm \hat{\mathcal{Z}}), \qquad \{\tilde{\mathcal{Q}}_{\alpha\dot{\beta}}, \tilde{\mathcal{Q}}^{\dagger\rho\dot{\lambda}}\} = \delta_{\alpha}{}^{\rho}\delta_{\dot{\beta}}{}^{\dot{\lambda}}(\mathcal{H} \pm \hat{\mathcal{Z}})$$
$$\{\tilde{Q}_{\dot{\alpha}\beta}, \tilde{Q}_{\dot{\rho}\lambda}\} = 0, \qquad \{\tilde{Q}_{\alpha\dot{\beta}}, \tilde{Q}_{\rho\dot{\lambda}}\} = 0. \tag{3.46}$$

(Note that each of the plus and minus signs in (3.45) is giving four independent supercharges and hence we need to consider both signs to capture the whole superalgebra.) Therefore the right-hand-side of the above superalgebra vanishes for eight (half of) supercharges if $\mathcal{H} = \hat{\mathcal{Z}}$.

4. Discussion

In a quest to enhance our understanding of dynamics of D3-branes in the plane-wave background we have constructed 1/4 BPS configurations of such D-brane using the light-cone

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Hamiltonian and the corresponding (dynamical) superalgebra. The BPS configurations we studied in this work all involve static electric field of the brane.

We first studied the BPS configuration corresponding to the electric field sourced by an electric dipole on the three sphere giant graviton, generalizing BIons to spherical branes. The electric dipole is composed of two opposite point charges Q placed on the North and South poles of the brane. In contrast to the flat brane BIons, due to non-linear Born-Infeld dynamics, the (double) spikes are capped off and do not extend to infinity. The details of the configuration, e.g. the length of the spikes or the size of their throats, are controlled by parameter $\lambda = Q/\mu$ (μ is the scale associated with the background plane-wave). We also discussed that besides the outward double spike configuration, we can also have inward spikes, spikes piercing through the three sphere giant (see the figure). The inward spike solution, however, only exists for $\lambda \leq 1/4$. Although we start with placing two point charges on the three sphere giant, and hence the corresponding electric field is singular, the shape of the brane is deformed in a specific manner such that the final configuration in the end is smooth with no singularity. This is important because in principle the DBI action is not capturing all the α'-corrections to the brane dynamics and there are higher order derivative α'-corrections to DBI [24]. For our solutions, however, higher derivative terms are subleading to the same α' order already present in the DBI.

It is of obvious interest to study and analyze similar double spike solutions from the dual $\mathcal{N} = 4$ SYM theory as well as the tiny graviton matrix theory [18, 22]. For the former, it is notable that the "Fat Magnons" of [26], which are bound states of "giant magnons" of [25] and giant gravitons, are indeed the same object as our double spike solutions. The fact that in the full Born-Infeld description the spikes turn out to have finite length and energy is compatible with the expectation of constructing open string excitations of the giants using the BMN type construction discussed in [10]. We hope to elaborate further on this question in upcoming publications.

Besides the double spike solutions we also studied squashed giant configurations and as showed there is a critical value for the electric field, $E = \mu$, where the shape of the giant becomes singular. It is desirable to understand better this critical electric field both from the brane theory and the dual $\mathcal{N} = 4$ SYM theory viewpoint.

We have also analyzed another class of 1/4 BPS configurations, the stringy solutions. As discussed these strings which are in fact deformations about the $X = 0$ vacuum of the theory, can be understood as follows. Each string is made out of (infinite number of) tiny electric dipoles, which is the absence of an external electric field, which due to the harmonic oscillator potential well provided by the background plane-wave, are all sitting on top of each other at $X = 0$. When the electric field is turned on, these dipoles are all aligned in the direction of the electric field and hence the tension of these strings are proportional to the electric field. In the directions orthogonal to the electric field, because of the harmonic oscillator potential coming from the background plane-wave metric, we are still dealing with point like objects. This picture is a very interesting one, suggesting that the strings on the background plane-wave are made out of "string bits" and the string bits in their own turn are tiny, dipole like three spheres and a fundamental string is in fact an (electric) flux tube. This dipole interpretation gives a realization of the string bit intuition coming from the BMN analysis. Moreover, this is also compatible with the picture developed in the tiny graviton matrix theory (TGMT) [18], according which the tiny three sphere branes, tiny gravitons, each carrying one unit of the light-cone momentum are the building blocks of the type IIB strings on the $AdS_5 \times S^5$ or the plane-wave backgrounds. The other interesting question regarding the string-like 1/4 BPS configurations discussed here is to compare them with the "giant magnon" [25] or the "fat magonon" [26] configurations, though after taking the Penrose limit.

In this work we only discussed 1/4 BPS configurations involving only electric field. As we showed, we have similar solutions in which the electric field is replaced by magnetic field (according to (2.15)). It is also possible to have 1/4 BPS configurations involving both static electric and magnetic fields, which we did not analyze here. Besides the 1/4 BPS solutions, we can have more less BPS (1/8 BPS) configurations with more general electric and magnetic fields. Analysis of these configurations is postponed to future works.

A. Polyakov form of the D_p-brane action

Dirac-Born-Infeld (DBI) action which describes dynamics of a D_p-brane can be put in another useful form by introducing an additional field on the world-volume, an independent world-volume metric. This form is useful in fixing the light-cone gauge (*cf.* footnote 2). DBI-action is presented by

$$S = -\frac{T}{g_s} \int d^{p+1}\sigma \sqrt{-\det\left(g_{\hat{\mu}\hat{\nu}} + F_{\hat{\mu}\hat{\nu}}\right)} \tag{A.1}$$

where $\hat{\mu} = 0, \ldots, p$. Recalling the symmetry and antisymmetry of $g_{\hat{\mu}\hat{\nu}}$ and $F_{\hat{\mu}\hat{\nu}}$, we can write

$$\left[\det(g_{\hat{\mu}\hat{\nu}} + F_{\hat{\mu}\hat{\nu}})\right]^{\frac{1}{2}} = \left[\det(g_{\hat{\mu}\hat{\nu}} - F_{\hat{\mu}\hat{\nu}})\right]^{\frac{1}{2}} = \left[\det(g_{\hat{\mu}\hat{\nu}})\det(g_{\hat{\mu}\hat{\nu}} - F_{\hat{\mu}\hat{\alpha}} g^{\hat{\alpha}\hat{\beta}} F_{\hat{\beta}\hat{\nu}})\right]^{\frac{1}{4}} \tag{A.2}$$

Next consider the action [27]

$$S' = -\frac{T'}{g_s} \int d^{p+1}\sigma \det(hg)^{\frac{1}{4}} \left[h^{\hat{\mu}\hat{\nu}}(g - F^2)_{\hat{\mu}\hat{\nu}} + \Lambda\right] \tag{A.3}$$

where h is a dynamical worldvolume metric and $F^2_{\hat{\mu}\hat{\nu}} = F_{\hat{\mu}\hat{\alpha}} g^{\hat{\alpha}\hat{\beta}} F_{\hat{\beta}\hat{\nu}}$. Upon eliminating the h field using its equation of motion, S' reduces to the DBI action. To see this we note that the equation of motion of the h field is obtained by setting the energy-momentum tensor, which by definition is the variation of the action with respect to the worldvolume metric, equal to zero. That is,

$$T_{\hat{\mu}\hat{\nu}} = \frac{\delta S'}{\delta h^{\hat{\mu}\hat{\nu}}} = (g - F^2)_{\hat{\mu}\hat{\nu}} - \frac{1}{4} h_{\hat{\mu}\hat{\nu}} \left[h^{\hat{\alpha}\hat{\beta}}(g - F^2)_{\hat{\alpha}\hat{\beta}} + \Lambda\right] = 0 \tag{A.4}$$

Λ can be identified taking the trace of the above equation:

$$\Lambda = \frac{3-p}{p+1} h^{\hat{\mu}\hat{\nu}}(g - F^2)_{\hat{\mu}\hat{\nu}}. \tag{A.5}$$

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(A.5) and (A.4) then yield

$$\det\left((g-F^2)_{\hat{\mu}\hat{\nu}}\right) = \det(h_{\hat{\mu}\hat{\nu}})\left[\frac{h^{\hat{\alpha}\hat{\beta}}(g-F^2)_{\hat{\alpha}\hat{\beta}}}{p+1}\right]^{p+1}. \tag{A.6}$$

Using the above and (A.2) we recover the DBI-action once we insert (A.6) into Polyakov action (A.3) (note that as is seen from (A.5) $h^{\hat{\mu}\hat{\nu}}(g-F^2)_{\hat{\mu}\hat{\nu}}$ is a constant and not a variable or field.) It is also notable that for the case of our interest, i.e. $p = 3$, Λ vanishes.

B. Derivation of the light-cone Hamiltonian in more detail

To obtain the light-cone Hamiltonian we note that due to the local diffeomorphism invariance the Hamiltonian $\tilde{\mathcal{H}}$

$$\tilde{\mathcal{H}} = \sum_\alpha \frac{\partial L}{\partial \dot{\Phi}_\alpha}\dot{\Phi}_\alpha - L \tag{B.1}$$

with $\Phi_\alpha \in \{X^+, X^-, X^I,\ A_0, A_r, \psi, \bar{\psi}\}$, should vanish for all physical configurations. Besides $\tilde{\mathcal{H}} = 0$, in the light-cone gauge one should also impose (2.8) on the physical configurations. In the light-cone gauge, after imposing (2.7) and (2.4) we have [17]

$$\begin{aligned}\tilde{\mathcal{H}} &= p^+\partial_\tau X^- + P_I\dot{X}^I + P^- + \frac{\partial L}{\partial F_{0r}}F_{0r} + \frac{\partial L}{\partial\dot{\psi}}\dot{\psi} + \frac{\partial L}{\partial\dot{\bar{\psi}}}\dot{\bar{\psi}} - L \\ &= 0\end{aligned} \tag{B.2}$$

where we have also imposed the U(1) gauge theory constraint

$$\partial_r P^r_E = 0\,, \tag{B.3}$$

and

$$P^I = \frac{\partial\mathcal{L}}{\partial\dot{X}_I} = -p^+\dot{X}^I \tag{B.4}$$

$$P^r_E = \frac{\partial\mathcal{L}}{\partial F_{0r}} = \frac{1}{g_s}\sqrt{-detN}N^{0r}. \tag{B.5}$$

In the above two equations (2.8), that is $N^{0r} = -N^{r0}$, has been used. From (B.2) we learn that

$$H_{lc} \equiv P^- = -\left[p^+\partial_\tau X^- + P_I\dot{X}^I + P^r_E F_{0r} + \frac{\partial L}{\partial\dot{\psi}}\dot{\psi} + \frac{\partial L}{\partial\dot{\bar{\psi}}}\dot{\bar{\psi}} - L\right] \tag{B.6}$$

We should now eliminate $\dot{\psi},\ \dot{X}^I,\ F_{0r}$ and $\partial_\tau X^-$ in favor of the canonical variables and the conjugate momenta. Let us start with $\partial_\tau X^-$ and recall the definition of $\det N$ which is

$$\det N = \det(N_{rs})(N_{00} - N_{0r}N^{rs}N_{s0}), \tag{B.7}$$

where

$$N_{00} = -2\partial_\tau X^- - \mu^2(X^I)^2 + (\dot{X}^I)^2 + 2i(\bar{\psi}\bar{\gamma}^-\partial_\tau\psi + \psi\bar{\gamma}^-\partial_\tau\bar{\psi}) - 4\mu\bar{\psi}\bar{\gamma}^-\Pi\psi \tag{B.8}$$

and N^{rs} is the inverse of N_{rs}, that is $N^{rs}N_{sp} = \delta^r_p$. It is important to note that $N^{00} \neq \frac{1}{N_{00}}$ because of the off-diagonal electric-magnetic fields. By definition we have

$$N^{00} = \frac{det(N_{rs})}{detN} = \frac{det(g_{rs}+F_{rs})}{detN} \tag{B.9}$$

The above two equations together with (2.9) lead to

$$N_{00} = -\left(\frac{1}{p^+ g_s}\right)^2 det(N_{rs}) + N_{0r}N^{rs}N_{s0} \tag{B.10}$$

Now we can eliminate $\partial_\tau X^-$ in (B.8) using (B.10) and $\dot{X}^I$ using (B.4). To eliminate F_{0r} for P^r_E we use (B.5) and recall that

$$N^{0r} = -\frac{\det(N_{rs})}{\det N} N^{sr}N_{0s},$$

yielding

$$P^r_E = \frac{1}{g_s}\frac{\det(N_{rs})}{\sqrt{-\det N}} N^{sr}N_{0s} = p^+ N^{rs}N_{s0}. \tag{B.11}$$

In the above we have also used the following identities

$$\begin{aligned} N^{0r} &= -\, N^{r0} \\ N^{sr}N_{0s} &= -\, N^{rs}N_{s0}, \end{aligned} \tag{B.12}$$

With the above the light-cone Hamiltonian (2.9) is obtained to be

$$\begin{aligned} P^- =& \frac{(P^I)^2}{2p^+} + \frac{(P^I_E)^2}{2p^+} + \frac{1}{2}\mu^2 p^+ (X^I)^2 + \frac{1}{2p^+ g_s^2} det(N_{rs}) + 2p^+\mu\bar{\psi}\bar{\gamma}^-\Pi\psi \\ &+ \frac{\mu}{6g_s}\epsilon^{rps}\big[\epsilon^{ijkl}X^i\partial_r X^j\partial_p X^k\partial_s X^l + \epsilon^{abcd}X^a\partial_r X^b\partial_p X^c\partial_s X^d \\ &- \partial_r X^I\partial_p X^J\bar{\psi}\gamma^{-IJ}\partial_s\psi\big] \end{aligned} \tag{B.13}$$

where

$$P^I_E = P^r_E\partial_r X^I. \tag{B.14}$$

For the special case of D3-brane $det(F_{rs}) = 0$ and therefore

$$\det(N_{rs}) = det(g_{rs}) + (B^I)^2, \tag{B.15}$$

where

$$B^I = B^r\partial_r X^I = -\frac{1}{\sqrt{2}}\epsilon^{rsp}\partial_r X^I F_{sp}. \tag{B.16}$$

Putting all these together we find the light-cone Hamiltonian density

$$\begin{aligned} H_{lc} =& \frac{(P^I)^2}{2p^+} + \frac{(P^I_E)^2}{2p^+} + \frac{1}{2}\mu^2 p^+ (X^I)^2 + \frac{1}{2p^+ g_s^2} det(g_{rs}) + \frac{1}{2p^+ g_s^2}(B^I)^2 + 2p^+\mu\bar{\psi}\bar{\gamma}^-\Pi\psi \\ &+ \frac{\mu}{6g_s}\big(\epsilon^{ijkl}X^i\{X^j,X^k,X^l\} + \epsilon^{abcd}X^a\{X^b,X^c,X^d\}\big) - \psi\gamma^{-IJ}\{X^I,X^J,\psi\}, \end{aligned} \tag{B.17}$$

which should be supplemented by (B.3).

To obtain H_{lc} given in (2.10) we need to move to the $\mathrm{SO}(4)\times\mathrm{SO}(4)$ representation for fermions. The details of which may be found in appendix C. Recall also that

$$\begin{aligned}\{F,G,K\} &= \epsilon^{rps}\partial_r F\ \partial_p G\ \partial_s K\\ \det(g_{rs}) &= \det(\partial_r X^I \partial_s X^I) = \frac{1}{3!}\{X^I,X^J,X^K\}\{X^I,X^J,X^K\}.\end{aligned}$$

C. Fermionic notations

C.1 Metsaev's fermionic notation

For completeness we summarize the fermionic notation which is used in section 2.1 [14]. Chiral representation are used for 32×32 matrices Γ in terms of 16×16 matrices γ

$$\Gamma^\mu = \begin{pmatrix} 0 & \gamma^\mu \\ \bar\gamma^\mu & 0 \end{pmatrix}, \quad \mu = 0,1,\ldots,9$$

$$\gamma^\mu\bar\gamma^\nu + \gamma^\nu\bar\gamma^\mu = 2\eta^{\mu\nu}\ , \qquad \gamma^\mu = (\gamma^\mu)^{\alpha\beta}\ , \qquad \bar\gamma^\mu = (\bar\gamma^\mu)_{\alpha\beta} \tag{C.1}$$

$$\gamma^\mu = (\mathbf{1},\gamma^I,\gamma^9)\ ,\ \ \bar\gamma^\mu = (-\mathbf{1},\gamma^I,\gamma^9)\ ,\ \ \alpha,\beta = 1,\ldots,16 \tag{C.2}$$

note that all γ^μ matrices are real and symmetric and

$$\begin{aligned}(\gamma^{\mu\nu})_\beta^{\ \alpha} &\equiv \frac{1}{2}(\gamma^\mu\bar\gamma^\nu)_\beta^{\ \alpha} - (\mu\leftrightarrow\nu)\\ (\bar\gamma^{\mu\nu})_\alpha^{\ \beta} &\equiv \frac{1}{2}(\bar\gamma^\mu\gamma^\nu)_\alpha^{\ \beta} - (\mu\leftrightarrow\nu)\\ \gamma^{+-} &= \gamma^0\gamma^9\end{aligned} \tag{C.3}$$

The 32-component positive and negative chirality spinor are decomposed in term of 16-component spinors as

$$\psi = \begin{pmatrix}\psi^\alpha \\ 0\end{pmatrix}\ ,\quad \theta = \begin{pmatrix} 0 \\ \theta_\alpha\end{pmatrix}$$

The complex Weyl spinor ψ is related to two real Majorana-Weyl spinors ψ^1 and ψ^2 by

$$\psi = \frac{1}{\sqrt{2}}(\psi^1 + i\psi^2)\quad ,\quad \bar\psi = \frac{1}{\sqrt{2}}(\psi^1 - i\psi^2) \tag{C.4}$$

The short-hand notation $\bar\psi\gamma^\mu\psi$ stands for $\bar\psi^\alpha\gamma^\mu_{\alpha\beta}\psi^\beta$ and alike for similar bi-fermions.

C.2 The $\mathrm{SO}(4)\times\mathrm{SO}(4)$ fermionic notation

The Dirac matrices in ten dimensions obey

$$\{\Gamma^\mu,\Gamma^\nu\} = 2g^{\mu\nu} \tag{C.5}$$

A convenient choice of basis for 32×32 matrices, which is denoted by Γ, is in term of 16×16 matrices γ

$$\Gamma^{+} = i \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}, \ \Gamma^{-} = i \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}, \ \Gamma^{I} = \begin{pmatrix} \gamma^{I} & 0 \\ 0 & -\gamma^{I} \end{pmatrix}, \ \Gamma^{11} = \begin{pmatrix} \gamma^{(8)} & 0 \\ 0 & -\gamma^{(8)} \end{pmatrix}$$

and the γ satisfy $\{\gamma^I, \gamma^J\} = 2\delta^{IJ}$ with δ^{IJ} the metric on the transverse space and $\Gamma^{\pm} = \frac{1}{\sqrt{2}}(\Gamma^0 \pm \Gamma^9)$. We may choose our ten dimensional, 32 components Majorana fermions ψ to satisfy

$$\Gamma^{+}\psi^{+} = 0 \ , \quad \Gamma^{-}\psi^{-} = 0 \tag{C.6}$$

and it is easily seen that

$$\psi^{+} = \begin{pmatrix} \psi^{+}_{\alpha} \\ 0 \end{pmatrix}, \ \psi^{-} = \begin{pmatrix} 0 \\ \psi^{-}_{\alpha} \end{pmatrix}, \ \alpha = 1, \ldots, 16.$$

where $\psi^{\pm}_{\alpha}$ can be thought as SO(8) Majorana fermions and the γ^I matrices as 16×16 SO(8) Majorana gamma matrices. Moreover, we have

$$\Gamma^{11}\psi^{+} = \begin{pmatrix} \gamma^{(8)}\psi^{+}_{\alpha} \\ 0 \end{pmatrix}, \ \Gamma^{11}\psi^{-} = \begin{pmatrix} 0 \\ -\gamma^{(8)}\psi^{-}_{\alpha} \end{pmatrix}$$

i.e. the ten dimensional chirality is related to eight dimensional SO(8) chirality.

Let us now consider type IIB theory on the plane-wave background. In this case we start with ten dimensional fermions of the same chirality. As stated in the above equations, $\psi^{\pm}_{\alpha}$ should have $\pm$ SO(8) chirality i.e.

$$(\gamma^{(8)}\psi^{\pm})_{\alpha} = \pm\psi^{\pm}_{\alpha} \tag{C.7}$$

By selecting $\gamma^{(8)} = diag(\mathbf{1}_8, -\mathbf{1}_8)$ the last equation is easily solved and hence

$$\psi^{+}_{\alpha} = \begin{pmatrix} \psi^{+}_{a} \\ 0 \end{pmatrix}, \ \psi^{-} = \begin{pmatrix} 0 \\ \psi^{-}_{\dot{a}} \end{pmatrix}, \ a, \dot{a} = 1, \ldots, 8.$$

where ψ^{+}_{a},$\psi^{-}_{\dot{a}}$ are Weyl-Majorana fermions. The gamma matrices can also be reduced to 8×8 representation, $\gamma^{I}_{a\dot{a}}$ and $\gamma^{I}_{\dot{a}a}$,

$$\gamma^{I} = \begin{pmatrix} 0 & \gamma^{I}_{a\dot{a}} \\ \gamma^{I}_{\dot{a}a} & 0 \end{pmatrix}, \quad a, \dot{a}, I = 1, \ldots, 8.$$

In the plane-wave background, due to the presence of RR form flux, the SO(8) is broken to SO(4) $\times$ SO(4). It is therefore better to adopt SO(4) $\times$ SO(4) representation for fermions. SO(4) Dirac fermion can be decomposed into two Weyl fermions ψ_{α} and $\psi_{\dot{\alpha}}$, $\alpha, \dot{\alpha} = 1, 2$. As usual for SU(2) fermion Weyl indices are lowered and raised by ϵ tensor

$$\psi_{\alpha} = \epsilon_{\alpha\beta}\psi^{\beta} \tag{C.8}$$

Hence our fermions have two indices which are labeled to two different SO(4) Weyl indices i.e.

$$\begin{aligned} \psi_a &\rightarrow \psi_{\alpha\beta}\,, \psi_{\dot\alpha\dot\beta} \\ \psi_{\dot a} &\rightarrow \psi_{\dot\alpha\beta}\,, \psi_{\alpha\dot\beta} \end{aligned} \tag{C.9}$$

where the first (second) indices are related to first (second) SO(4). The Weyl indices are lowered and raised by two ϵ tensor

$$\psi^{\dagger}_{\alpha\beta} \equiv \epsilon_{\alpha\rho}\epsilon_{\beta\lambda}\psi^{\dagger\rho\lambda} \quad , \quad \psi^{\dagger}_{\dot\alpha\dot\beta} \equiv \epsilon_{\dot\alpha\dot\rho}\epsilon_{\dot\beta\dot\lambda}\psi^{\dagger\dot\rho\dot\lambda} \tag{C.10}$$

where

$$(\psi_{\alpha\beta})^{\dagger} = \psi^{\dagger\alpha\beta}, \quad (\psi_{\dot\alpha\dot\beta})^{\dagger} = \psi^{\dagger\dot\alpha\dot\beta} \tag{C.11}$$

$$(\psi^{\dagger}_{\alpha\beta})^{\dagger} = \psi^{\alpha\beta}, \quad (\psi^{\dagger}_{\dot\alpha\dot\beta})^{\dagger} = \psi^{\dot\alpha\dot\beta} \tag{C.12}$$

We also choose a proper basis for $\gamma^I_{a\dot a}$ which is

$$\gamma^I_{a\dot a} = (\gamma^i_{a\dot a}, \gamma^a_{a\dot a}) \tag{C.13}$$

where

$$\gamma^i_{a\dot a} = \begin{pmatrix} 0 & (\sigma^i)_{\alpha\dot\beta}\delta^{\theta}_{\rho} \\ (\sigma^i)^{\dot\alpha\beta}\delta^{\dot\theta}_{\dot\rho} & 0 \end{pmatrix} \quad , \quad \gamma^i_{\dot a a} = \begin{pmatrix} 0 & (\sigma^i)_{\alpha\dot\beta}\delta^{\dot\theta}_{\dot\rho} \\ (\sigma^i)^{\dot\alpha\beta}\delta^{\theta}_{\rho} & 0 \end{pmatrix}$$

and

$$\gamma^a_{a\dot a} = \begin{pmatrix} -\delta^{\beta}_{\alpha}(\sigma^a)_{\rho\dot\theta} & 0 \\ 0 & \delta^{\dot\beta}_{\dot\alpha}(\sigma^a)_{\dot\rho\theta} \end{pmatrix} \quad , \quad \gamma^a_{\dot a a} = \begin{pmatrix} -\delta^{\beta}_{\alpha}(\sigma^a)^{\dot\rho\theta} & 0 \\ 0 & \delta^{\dot\beta}_{\dot\alpha}(\sigma^a)^{\rho\dot\theta} \end{pmatrix}$$

with

$$(\sigma^i)_{\alpha\dot\alpha} = (\sigma^1, \sigma^2, \sigma^3, \mathbf{1})_{\alpha\dot\alpha} \tag{C.14}$$

In performing the superalgebra analysis we have used the following σ matrix identities

$$(\sigma^a)_{\alpha\dot\alpha} = (\sigma^1, \sigma^2, \sigma^3, \mathbf{1})_{\alpha\dot\alpha} \tag{C.15}$$

$$\big((\sigma^i)_{\alpha}{}^{\dot\beta}\big)^{\dagger} = (\sigma^i)_{\dot\beta}{}^{\alpha}, \qquad \big((\sigma^a)_{\alpha}{}^{\dot\beta}\big)^{\dagger} = (\sigma^a)_{\dot\beta}{}^{\alpha} \tag{C.16}$$

$$\big((\sigma^i)_{\dot\alpha}{}^{\beta}\big)^{\dagger} = (\sigma^i)_{\beta}{}^{\dot\alpha}, \qquad \big((\sigma^a)_{\dot\alpha}{}^{\beta}\big)^{\dagger} = (\sigma^a)_{\beta}{}^{\dot\alpha} \tag{C.17}$$

$$\big((\sigma^{ij})_{\alpha}{}^{\beta}\big)^{\dagger} = -(\sigma^{ij})_{\beta}{}^{\alpha}, \qquad \big((\sigma^{ab})_{\alpha}{}^{\beta}\big)^{\dagger} = -(\sigma^{ab})_{\beta}{}^{\alpha} \tag{C.18}$$

$$\big((\sigma^{ij})_{\dot\alpha}{}^{\dot\beta}\big)^{\dagger} = -(\sigma^{ij})_{\dot\beta}{}^{\dot\alpha}, \qquad \big((\sigma^{ab})_{\dot\alpha}{}^{\dot\beta}\big)^{\dagger} = -(\sigma^{ab})_{\dot\beta}{}^{\dot\alpha} \tag{C.19}$$

$$(\sigma^i)_{\alpha}{}^{\dot\beta} = \epsilon_{\alpha\rho}(\sigma^i)^{\rho\dot\beta} = (\sigma^i)_{\alpha\dot\rho}\epsilon^{\dot\rho\dot\beta}, \qquad (\sigma^a)_{\alpha}{}^{\dot\beta} = \epsilon_{\alpha\rho}(\sigma^a)^{\rho\dot\beta} = (\sigma^a)_{\alpha\dot\rho}\epsilon^{\dot\rho\dot\beta} \tag{C.20}$$

$$(\sigma^{ij})_{\alpha}{}^{\beta} = \epsilon_{\alpha\rho}(\sigma^{ij})^{\rho\beta} = (\sigma^{ij})_{\alpha\rho}\epsilon^{\rho\beta}, \quad (\sigma^{ab})_{\alpha}{}^{\beta} = \epsilon_{\alpha\rho}(\sigma^{ab})^{\rho\beta} = (\sigma^{ab})_{\alpha\rho}\epsilon^{\rho\beta} \tag{C.21}$$

Additional discussion about notation can be found in [13].

D. BPS equation

The BPS equations (3.2) and (3.3) are equations relating ϵ and $\epsilon^\dagger$. Let us first focus on (3.2), which can be written as

$$(\tilde{X}^i\sigma^i)^{\dot{\alpha}}{}_{\rho}\epsilon_{\dot{\alpha}\lambda} = (\tilde{\Pi}^i\sigma^i)^{\dot{\alpha}}{}_{\rho}\epsilon^\dagger_{\dot{\alpha}\lambda}, \tag{D.1}$$

where

$$\Pi^i = \frac{B^i}{g_s} + iP^i_E\ .$$

Next we note that

$$(\tilde{X}^i\sigma^i)^{\dot{\alpha}}{}_{\rho}(\tilde{X}^j\sigma^j)^{\rho}{}_{\dot{\beta}} = |\tilde{X}|^2\delta^{\dot{\alpha}}{}_{\dot{\beta}} \tag{D.2}$$

and

$$(\Pi^i\sigma^i)^{\dot{\alpha}}{}_{\rho}(\bar{\Pi}^j\sigma^j)^{\rho}{}_{\dot{\beta}} = \Pi^i\bar{\Pi}^i\delta_{\dot{\alpha}}{}^{\dot{\beta}} + \Pi^i\bar{\Pi}^j(\sigma^{ij})^{\dot{\alpha}}{}_{\dot{\beta}}. \tag{D.3}$$

Using the above two identities (D.1) leads to

$$\epsilon_{\dot{\beta}\lambda} = \mathcal{R}^{\dot{\alpha}}{}_{\dot{\beta}}\epsilon^\dagger_{\dot{\alpha}\lambda} = \frac{\tilde{X}^j\Pi^i}{|\tilde{X}|^2}\big((\sigma^{ij})^{\dot{\alpha}}{}_{\dot{\beta}} + \delta^{ij}\delta^{\dot{\alpha}}{}_{\dot{\beta}}\big)\epsilon^\dagger_{\dot{\alpha}\lambda}\,, \tag{D.4}$$

and

$$\tilde{X}^i\bar{\Pi}^j\big((\sigma^{ij})^{\dot{\alpha}}{}_{\dot{\beta}} + \delta^{ij}\delta^{\dot{\alpha}}{}_{\dot{\beta}}\big)\epsilon_{\dot{\alpha}\lambda} = \Pi^i\bar{\Pi}^j\big((\sigma^{ij})^{\dot{\alpha}}{}_{\dot{\beta}} + \delta^{ij}\delta^{\dot{\alpha}}{}_{\dot{\beta}}\big)\epsilon^\dagger_{\dot{\alpha}\lambda}\ . \tag{D.5}$$

Taking the complex conjugate of (D.4),

$$\epsilon^\dagger_{\dot{\beta}\lambda} = \frac{\tilde{X}^i\bar{\Pi}^j}{|\tilde{X}|^2}\big((\sigma^{ij})^{\dot{\alpha}}{}_{\dot{\beta}} + \delta^{ij}\delta^{\dot{\alpha}}{}_{\dot{\beta}}\big)\epsilon_{\dot{\alpha}\lambda} \tag{D.6}$$

and substituting the r.h.s. of (D.6) into the L.H.S of (D.5) we obtain

$$\left((|\tilde{X}|^2 - |\Pi|^2)\delta^{\dot{\alpha}}{}_{\dot{\beta}} - \Pi^i\bar{\Pi}^j(\sigma^{ij})^{\dot{\alpha}}{}_{\dot{\beta}}\right)\epsilon^\dagger_{\dot{\alpha}\lambda} = \mathcal{M}^{\dot{\alpha}}{}_{\dot{\beta}}\epsilon^\dagger_{\dot{\alpha}\lambda} = 0 \tag{D.7}$$

where $|\Pi|^2 = \Pi^i\bar{\Pi}^i$. One can exactly follow the same computations for (3.3) to obtain

$$\epsilon_{\beta\dot{\lambda}} = \mathcal{R}^{\alpha}{}_{\beta}\epsilon^\dagger_{\alpha\dot{\lambda}} = \frac{\tilde{X}^j\Pi^i}{|\tilde{X}|^2}\big((\sigma^{ij})^{\alpha}{}_{\beta} + \delta^{ij}\delta^{\alpha}{}_{\beta}\big)\epsilon^\dagger_{\alpha\dot{\lambda}}$$

and then

$$\left((|\tilde{X}|^2 - |\Pi|^2)\delta^{\alpha}{}_{\beta} - \Pi^i\bar{\Pi}^j(\sigma^{ij})^{\alpha}{}_{\beta}\right)\epsilon^\dagger_{\alpha\dot{\lambda}} = \mathcal{M}^{\alpha}{}_{\beta}\epsilon^\dagger_{\alpha\dot{\lambda}} = 0 \tag{D.8}$$

$\mathcal{M}$ as a 2×2 matrix can have one or two zero eigenvalue which support 1/16 or 1/8 BPS configurations for each of the two equations. Therefore, we can have 1/4 or 1/8 BPS configurations.

- 1/4 *BPS configurations*

  The first one takes place when $\mathcal{M}^{\dot\alpha}{}_{\dot\beta} \equiv 0$ and then

$$\begin{aligned} |\tilde{X}|^2 - |\Pi|^2 &= 0 \\ \Pi^{[i}\bar{\Pi}^{j]} &= 0 \end{aligned} \tag{D.9}$$

  Thus BPS equations have three different options:

  (i) $P_E \neq 0, B = 0 \to |\tilde{X}|^2 = |P_E|^2$

  (ii) $P_E = 0, B \neq 0 \to |\tilde{X}|^2 = \frac{1}{g_s^2}|B|^2$

  (iii) $P_E \neq 0, B \neq 0 \to |\tilde{X}|^2 = |\bar{\Pi}|^2$ *provided that* $P_E^i B^j = P_E^j B^i$

  The third case corresponds to parallel electric and magnetic fields because

$$B^j = \frac{B \cdot P_E}{P_E^2} P_E^j \ . \tag{D.10}$$

- 1/8 *BPS configurations*

  If $\mathcal{M}$ has only one zero eigenvalue we obtain 1/8 BPS configuration. In order $\mathcal{M}$ to have one zero eigenvalue

$$(|\tilde{X}|^2 - |\Pi|^2)^2 + \Pi^i\bar{\Pi}^j\Pi^{[i}\bar{\Pi}^{j]} = 0. \tag{D.11}$$

  It is obvious that when $\Pi^i \| \bar{\Pi}^i$ the second term vanishes and then this equation recover 1/4 BPS equation.

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